SOME RESULTS OF AN INVESTIGATION OF THE KINETICS OF SEPARATION AND MIXING OF DISPERSED MATERIALS

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This paper gives the results of an investigation of the kinetics of separation and mixing of dispersed materials due to vibration or a flow of liquid or gas in application to mineral enrichment and screen separation of grain products and other materials.

We imagine a fluidized bed of loose material containing particles which differ in their properties (geometric, mechanical, etc.). The position of a particle of any component of the mixture (small or large, heavy or light) is defined by the coordinate z, measured from the bottom of the bed.

In a fluidized bed z(t) will undergo random increases, and its variation will be regarded as a random Markovian process and interpreted as a linear random walk of the image point along the z axis. We will denote the probability density of the investigated process by w(t, z). Then wdz is the probability that the wandering particle is in the region (z, z + dz) at instant t.

By the probability we mean the number of favorable results of the random process. However, we can imagine numerous outcomes occurring simultaneously, i.e., we can picture a large number of independently moving particles instead of one moving particle. Then this probability gives the fraction of the total number of particles found in the prescribed region, i.e., wdz gives the relative number of particles located at instant t in the region (z, z + dz). Hence, the probability density has the sense of the relative concentration of image points and can be physically interpreted as the relative concentration of the particular component of the mixture.

The probability density of a one-dimensional Markovian process satisfies the Kolmogorov-Fokker-Planck differential equation

$$\frac{\partial w(t, z)}{\partial t} = -\frac{\partial}{\partial z}[c(t, z)w(t, z)] + \frac{1}{2}\frac{\partial^2}{\partial z^2}[b(t, z)w(t, z)], \qquad (1)$$

which describes the forced quasi-diffusion of image points.

The stochastic coefficient c(t, z) has the sense of the velocity of ordered motion of the particles under the action of an external field (gravitational, hydrodynamic, for instance). The coefficient b(t, z) is a measure of the disorder of the motion and has the sense of the coefficient of quasi-diffusion of the particles. If the process is homogeneous in space, then the stochastic coefficients may depend only on the time or, in particular, be constant. The quasi-diffusion coefficient b in this case depends on the size of the particles and the effective viscosity of the fluidized medium and characterizes the mobility of the particles in the bed.

If the particles of the mixture differ in density, the coefficient c will be proportional to the difference in densities of the particles, and, if this difference is small, the gravitational flow of the particles will cease to affect the transfer process.

In terms of random walk theory the last assumption corresponds to acceptance of the equiprobable motion of the particles in both directions. In the case where $c \neq 0$, i.e., when the external field has an organizing effect, the motion of the particles will be asymmetrical, more moving in one direction than the other.

If the process is due to a flow of water or air, the stochastic coefficients b and c will largely depend on the velocity of the fluidizing stream.

The literature of fluidization [1, 2] gives some information about the effective viscosity of a bed and the speeds of settling of the particles. This can be used to explain the relationships between the stochastic coefficients and the acting factors. Keeping within the framework of the stochastic description of the problem we will assume that the coefficients b and c are known, say, from experiment.

The solution of the differential equation (1) must be consistent with the boundary and initial conditions corresponding to the process.

By calculating the probability density w(t, z) we can solve several problems entailing the determination of the indices of mixing and separation processes. For instance, the obtained relationship w = w(t, z) gives the law of variation of the relative concentration. In the case of mixing of loose materials it will be possible to solve not only the problem of producing a mixture of maximum possible homogeneity, but also the separation problem—to separate the mixture according to the properties of the particles.

In the case of screen separation, and also in several industrial processes effected in a fluidized bed, the particles leave the bed—they pass through the holes in the screen or are carried off into the space above the bed. The indices of these processes are the extraction or entrainment, defined as the relative number of particles which have left the bed by the instant t. The extraction or entrainment can be defined by a "probability quantity" ε carried through the screen or the surface of the bed:

$$\varepsilon = \int_0^t \left[-\frac{1}{2}\frac{\partial}{\partial z}(bw) + cw\right]dt, \quad z = 0 \quad (z = h). \quad (2)$$

This expression gives the probability that a particle will leave the bed by the instant t. It is obvious that

$$1-\varepsilon = 1-\int_0^t \left[-\frac{1}{2}\frac{\partial}{\partial z}(bw)+cw\right]dt,$$

$$z=0 \quad (z=h)$$

gives the probability that the particle will remain in the bed during the time t, or the relative number of particles still in the bed at this time.

We will illustrate the foregoing by examples of calculation of the indices of mixing and screen separation.

Mixing. Let the initial distribution of particles in the mixer be specified by a delta function $w(0, z) = \delta(z - h)$. This is the distribution for the case where the mixture component is delivered to the surface of the bed. We define the evolution of this initial distribution by using Eq. (1) with constant stochastic coefficients

$$\frac{\partial w}{\partial t} = \frac{b}{2}\frac{\partial^2 w}{\partial z^2} + c\frac{\partial w}{\partial z}, \qquad (3)$$

in which c is the velocity of induced transfer of the particles towards the bottom of the bed.

As a boundary condition we assume that

$$\frac{b}{2}\frac{\partial w}{\partial z} + cw = 0$$

when z = 0 and z = h.

This condition for total reflection of the randomly moving particles from the boundaries of the region means that, on attaining these boundaries, the particles will subsequently take part in the mixing process.

The solution of Eq. (3) with these conditions will be

$$\bar w = \frac{w}{w_p} = 4\bar h\,\frac{\exp(-4\bar h\bar z)}{1-\exp(-4\bar h)} + \sum_{m=1}^{\infty} \times \left[2(-1)^m \exp[-2\bar h(\bar z-1)]\left(\cos m\pi\bar z - \frac{2\bar h}{m\pi}\sin m\pi\bar z\right)m^2\pi^2\right] \times \left[4\bar h^2+m^2\pi^2\right]^{-1} \times \exp\left[-\left(\frac{m^2\pi^2}{4\bar h^2}+1\right)\bar t\right]. \qquad (4)$$

The obtained equation for the kinetics of mixing relates the relative concentration of dynamically "heavy" ($\bar h > 0$) particles, and "light" particles ($\bar h < 0$) to the two process parameters $\bar t$ and $\bar h$, which depend on the two experimental coefficients b and c characterizing the dispersion of the particles and the intensity of the external field.

When particles are mixed in the absence of an external field, i. e., when the particles have the same mechanical properties, the velocity of induced transfer c = 0. In this case, when $\bar h = 0$ we obtain from (4)

$$\bar w = 1 + \sum_{m=1}^{\infty} 2(-1)^m \cos m\pi\bar z \exp\left(-\frac{b^2 m^2\pi^2}{h^2}t\right). \qquad (5)$$

According to (5), with the elapse of time the composition of the mixture evens out over its thickness and when $(\pi^2 b/2h^2)t \geq 4$ the concentration of the mixture differs from that for a uniform distribution by not more than 5%.

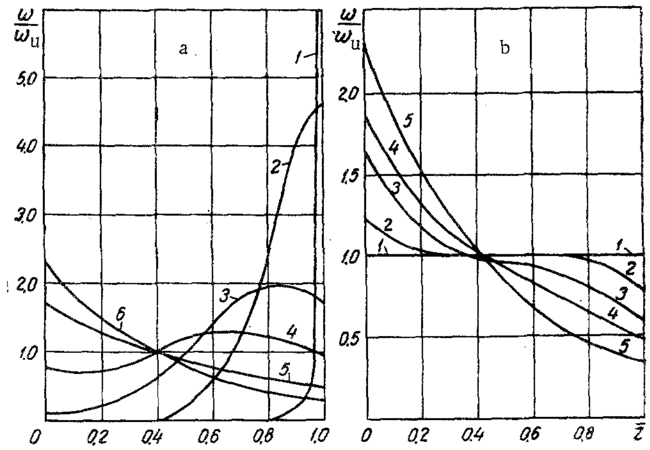

Fig. 1. Distribution of relative concentration of heavy particles after different times of mixing for cases of a deltoid initial distribution (a) and a uniform initial distribution (b) with h = 0.5: 1) t = 0, 2) 0.01; 3) 0.05; 4) 0.1; 5) 0.2; 6) ∞.

As distinct from this case, a uniform distribution of particles in the bed cannot be obtained in the case of mixing of dynamically heterogeneous particles differing in density and resistance to motion (Fig. 1a).

The heavy particles located on the surface of the bed at the start of the process penetrate into the bed and, finally, attain a limiting distribution for the particular $\bar h$.

Although a uniform distribution cannot be attained, there is at a certain instant $\bar t$ (in this case $\bar t \approx 0.12$) an "optimum" particle distribution. Equation (4) can be used for optimization of the process. If the quality of the mixing process is characterized by the mean square deviation from the uniform particle distribution

$$N = \frac{1}{h}\int_0^h \left(\frac{w-w_p}{w_p}\right)^2 dz,$$

the minimization of this index will give the optimum value of the parameter $\bar t$ for a given $\bar h$.

The equations for the kinetics of the process can be obtained for any other specified initial distributions. For example, in the problem of the demixing of particle mixtures, which is an important factor in gravity enrichment processes, the case of a uniform initial particle distribution in the layer (Fig. 1b) can be of interest.

For this case we can obtain the following equation for the kinetics of demixing of the mixture:

$$\bar w = 4\bar h\,\frac{\exp(-4\bar h\bar z)}{1-\exp(-4\bar h)} +$$

$$+\sum_{m=1}^{\infty}\frac{8m^2\pi^2\bar{h}\,[1-(1)^m\exp 2\bar{h}]\exp(-2\bar{h}\bar{z})}{(4\bar{h}^2+m^2\pi^2)^2}\times$$
$$\times\left(\frac{2\bar{h}}{m\pi}\sin m\pi\bar{z}-\cos m\pi\bar{z}\right)\times$$
$$\times\exp\left[-\left(\frac{m^2\pi^2}{4\bar{h}^2}+1\right)\bar{t}\right].$$

With the passage of time the initially uniform distribution again becomes a limiting one. The "light" particles "float up" to the surface of the bed in the separation process, and the heavy particles settle to the bottom.

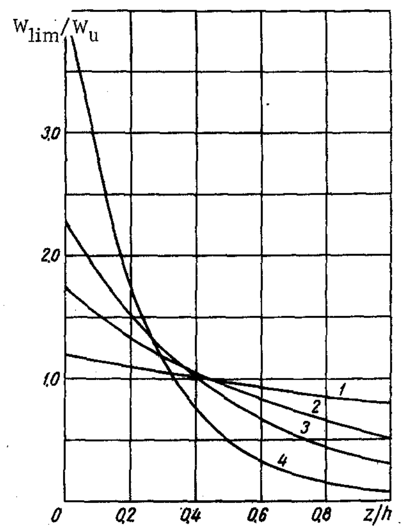

Fig. 2. Limiting distributions of heavy particles over thickness of bed: 1) h = 0.1, 2) 0.3; 3) 0.5; 4) 1.0.

Irrespective of the initial conditions, the limiting distributions of heavy particles over the thickness of the bed for different $\bar{h}$ (Fig. 2) are such that the mirror reflection of these curves gives the curves corresponding to the mixing of light particles.

If the material is fed continuously into the mixer during the mixing process the length of time spent by the mixture in the apparatus can be related to the adjustable and kinematic parameters and the feed rate. The coefficients c and b, which are required for calculation of the mixing indices, can be determined by two experiments.

In the presented sample calculation we ignored possible mixing of the load in the apparatus due to the action of agitators of any kind or convection currents produced in apparatuses with a fluidized gas.

It is to be hoped that within the framework of the expounded theory it will be possible to take into consideration macromixing of the load in the mixer and to avoid the assumption of uniformity of the process and constancy of the coefficients b and c, or a one-dimensional treatment of the problem.

Separation. A "vibro-separated" mixture consists of particles of different shapes, sizes, and densities. The efficiency of separation of the mixture in screen separation processes can be estimated in most cases by the extraction of small heavy particles under the screen and the possible extraction of small light particles contaminating the main product.

On the assumption of constancy of the stochastic coefficients the extraction can be calculated from the formula

$$\varepsilon=\int_0^t\left[\frac{b}{2}\frac{\partial w}{\partial z}+cw\right]dt,\quad z=0. \tag{6}$$

We determine the probability density w(t, z) from the equation

$$\frac{\partial w}{\partial t}=\frac{b}{2}\frac{\partial^2 w}{\partial z^2}+c\frac{\partial w}{\partial z} \tag{7}$$

with appropriate initial and boundary conditions.

Assume that at the initial instant w = w(0, z). For practically feasible conditions of loading of the material there is a uniform distribution over the thickness of the bed

$$w(0,\ z)=1/h$$

and a deltoid distribution

$$w(0,\ z)=\delta(z-h).$$

As one boundary condition we use the condition of zero particle flux through the upper boundary of the bed, i. e.,

$$\frac{b}{2}\frac{\partial w}{\partial z}+cw=0 \text{ when } z=h.$$

Taking into account that some or all of the small particles which reach the surface of the screen drop out of the bed, we will assume that the particle flux through the surface of the screen is proportional to their concentration at this surface, i. e.,

$$\frac{b}{2}\frac{\partial w}{\partial z}+cw=kw \text{ when } z=0.$$

This assumption, which has been experimentally confirmed in numerous investigations of the separation and sieving of loose materials in a shallow bed, was arrived at in [3] from a stochastic description of the passage of particles through a screen. The sifting coefficient k, as shown in this paper, is proportional to the initial particle concentration and the difference in the sizes of the holes and particles.

When k = ∞ (w = 0) and z = 0 unobstructed sifting occurs; when k = 0 there is no extraction of particles under the screen and mixing takes place.

With these conditions we obtain

$$\varepsilon=1-\sum_{m=1}^{\infty}a_m(\bar{h},\ \alpha)\exp\left[-\left(\frac{\rho_m^2}{4\bar{h}^2}+1\right)\bar{t}\right], \tag{8}$$

where $\bar{h}$ = ch/2b; α = 2k/c; ρ_m are the positive roots of the transcendental equation

$$\operatorname{tg}\rho_m = \frac{2\alpha\bar{h}\rho_m}{\rho_m^2 + 4\bar{h}(1-\alpha)}. \qquad (9)$$

The form of the function $a_m(\bar{h}, \alpha)$ depends on the initial particle distribution and the nature of the boundary conditions. Thus, for the case of unobstructed precipitation of particles from the bed ($k = \infty$, $\alpha = \infty$, $\rho_m \operatorname{ctg} \rho_m = -2\bar{h}$) and a uniform initial distribution we obtain the expression

$$a_m = \frac{2\rho_m(\rho_m + 4\bar{h}\sin\rho_m \exp 2\bar{h})}{(\rho_m^2 + 4\bar{h}^2)(\rho_m^2 + 4\bar{h}^2 + 2\bar{h})}, \qquad (10)$$

and for a deltoid distribution

$$a_m = \frac{2\rho_m \sin\rho_m \exp 2\bar{h}}{\rho_m^2 + 4\bar{h}^2 + 2\bar{h}}. \qquad (11)$$

In addition to calculation of the screen separation indices the presented formulas can be used to determine particle entrainment from a fluidized bed, and also the probability of the residence time of a particle in the bed. The expression

$$1 - \varepsilon = \sum_{m=1}^{\infty} a_m \exp\left[-\left(\frac{\rho_m^2}{4\bar{h}^2} + 1\right)\bar{t}\right] \qquad (12)$$

gives the probability of a particle remaining in the bed until the instant t, or the relative number of particles remaining in the bed at this time.

If at the initial instant the particles are uniformly distributed throughout the thickness of the bed, then a_m is given by formula (10); in the case of a deltoid distribution a_m is given by (11). It is assumed in this case that light particles are entrained through the upper boundary of the bed or heavy particles through the lower boundary. In particular, when formulas (10) and (11) are used, it is assumed that particles which reach the boundary of the bed leave it without obstruction.

The above calculation ignores the effect of the separation space on the entrainment of the solid phase from the bed. Within the framework of the propounded theory we should be able to allow for the return of particles from the space above the bed in calculating the entrainment.

The relationships for the entrainment and the time spent by particles in the bed, derived from the curves shown in Fig. 3, agree qualitatively with experimental results [1].

In the case of separation of mixtures of particles of the same density in the absence of a flow under the screen we can assume that the stochastic coefficient $c = 0$, $\bar{h} = 0$.

For this case, which corresponds to sieving, we obtain for a deltoid initial particle distribution

$$\varepsilon = 1 - \sum_{m=1}^{\infty} \frac{2\sin\rho_m(4\bar{h}_1^2 + \rho_m^2)}{\rho_m(\rho_m^2 + 4\bar{h}_1^2 + 2\bar{h}_1)} \times$$

$$\times \exp\left(-\frac{\rho_m^2}{4H^2}\right), \qquad (13)$$

where ρ_m is determined from the equation $\rho_m \operatorname{tg} \rho_m = 2\bar{h}_1$ and the symbols

$$\bar{h}_1 = \bar{h}k/b, \quad H = h/\sqrt{2bt}$$

are used.

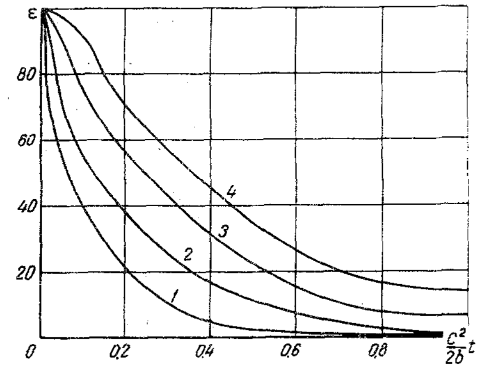

Fig. 3. Curves of "entrainment" ε, %, of particles from bed for the case of a uniform initial distribution: 1) $\bar{h}$ = 0.4; 2) 0.6; 3) 0.8; 4) 1.0.

If the relationship between the size of the particles and the holes in the screen is such that sieving is unobstructed, we can regard the surface of the screen as totally absorbing and put

$$k = \infty, \quad \bar{h}_1 = \infty, \quad \rho_m = (2m-1)\pi/2.$$

Then we obtain for a deltoid initial distribution

$$\varepsilon = 1 - \frac{4}{\pi}\sum_{m=1}^{\infty} \frac{(-1)^m}{2m-1} \exp\left[-\frac{(2m-1)^2\pi^2}{16H^2}\right], \qquad (14)$$

and for a uniform distribution

$$\varepsilon = 1 - \frac{8}{\pi^2}\sum_{m=1}^{\infty} \frac{1}{(2m-1)^2} \exp\left[-\frac{(2m-1)^2\pi^2}{16H^2}\right]. \qquad (15)$$

These formulas connect the extraction with one dimensionless parameter $H = h/(2bt)^{1/2}$, which depends only on the coefficient b. In the case of continuous feed of material the stay time of the particles on the screen depends on the screen length L and the mean feed rate V, which depend on adjustable parameters and kinematic conditions—the angles of the apparatus and the direction of the vibrations of the screen, their frequency, and amplitude.

Taking the volume output as $Q = hVB$, we obtain expressions for the parameter H:

$$H = h/\sqrt{2bt} = h \Big/ \sqrt{2b\frac{L}{V}} = Q/B\sqrt{2bLV}.$$

Thus, the indices of the efficiency of the process are related to the parameters of the mechanical regime and the load and, hence, the process can be controlled.

The results of the sample calculations and the revealed relationships for the separation of loose mate-

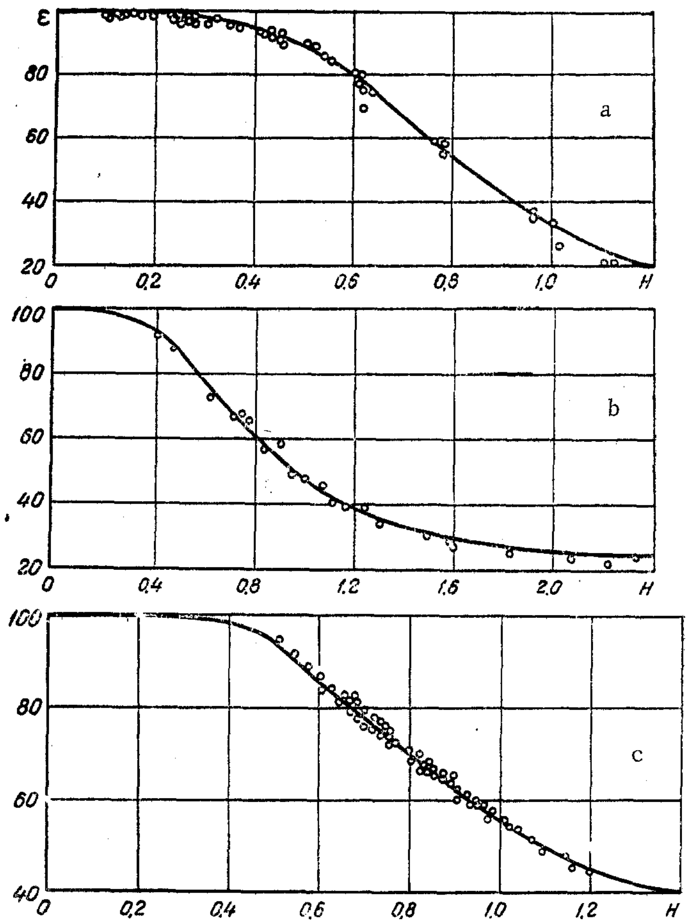

Fig. 4. Comparison of results of calculations of ε, %, with experiment: a) glass spheres; b) corn; c) buckwheat.

rials have been confirmed by experiments on screening and precipitation of minerals and nonmetalliferous materials, separation of grain products, and other loose materials.

As an example Fig. 4 shows the results of a comparison of the theoretical and experimental data for the separation of a two-component mixture of glass spheres of diameter 2000 μ (90 and 95%) and 800 μ (10 and 5%) on a laboratory screen (Rotap) with 1200 μ mesh. The small particles were loaded in batches onto the surface of the bed and the height of the bed and the duration of the process were varied. The separation coefficient b, determined from one experiment, was 0.8 cm^2/sec. The experimental points lie on the curve calculated from formula (14).

This figure shows the results of a comparison of the theoretical and experimental data for the separation of corn [4] and buckwheat [5]. In both cases the adjustable, kinematic, and load parameters of the machines were varied in a wide range.

NOTATION

b and c are the stochastic coefficients; w is the probability density; h is the height of bed; ε is the extraction or entrainment; $\bar{z} = z/h$, $\bar{h} = (c/2b)h$, and $\bar{t} = (c^2/2b)t$ are the dimensionless height of section, thickness of bed, and duration of process; $w_u = 1/h$ is the probability density for a uniform particle distribution over the thickness of the bed; N is the mean-square deviation from uniform particle distribution; k is the sifting coefficient; $\bar{h}_1 = (k/b)h$ and $H = h/(2bt)^{1/2}$ are the process parameters; L and B are the length and breadth of screen; V is the mean feed rate; Q is the volume output.

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